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1

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DOUBLE BETA DECAY: A THEORETICAL OVERVIEW

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1 INTRODUCTION

Now that Elliott, Hahn, and Moe[1] have observed the two-neutrino mode of double beta decay in the laboratory and the LBL-Santa Barbara collaboration[2] has set a limit of almost 10^{24} years on the no-neutrino mode in ^{76}Ge, I want to consider what the next steps in this field should be. How do we build on the truly significant advances that have been made in the past five years?

Although the principal motivation for studying double beta decay[3] comes from particle physics, the setting occurs in even-even nuclei and so the practical problems, especially the theoretical ones, are problems of nuclear physics. Progress in solving them will obviously lead to progress in the field itself. This is the aspect of double beta decay that I wish to emphasize in my talk today.

The three double beta decay modes of experimental interest are two-neutrino decay, no-neutrino decay, and Majoron decay. Two-neutrino decay, in which the nucleus (A, Z) transforms into the nucleus $(A, Z+2)$ and emits two electrons and two electron-type anti-neutrinos,

$$(A, Z) \rightarrow (A, Z+2) + 2e^- + 2\bar{\nu}_e \tag{1}$$

is expected to occur in the standard model as a second-order effect in the Fermi coupling constant G_F. It is mainly a test of nuclear physics, that is our ability to calculate the nuclear transition matrix element and the half-life for the process, but the standard model does make predictions about such properties as angular distributions which ought to be checked.

No-neutrino decay, in which the nucleus (A, Z) again transforms into the nucleus $(A, Z+2)$, this time with the emission of two electrons but no neutrinos,

$$(A, Z) \rightarrow (A, Z+2) + 2e^- \tag{2}$$

is the mode of the greatest and most fundamental interest. Observation of it would imply that: (1) lepton number is not conserved; and (2) at least one neutrino must be a Majorana particle with non-vanishing mass. If one neutrino is a Majorana particle then it is likely that all neutrinos are of this character. No-neutrino decay is not expected to occur in the standard model and so observing it would take us into the realm of 'Physics Beyond the Standard Model'.

The Majoron decay mode arises in a specific model for lepton nonconservation[4] in which lepton number is regarded as a global symmetry instead of a gauge one. Spontaneous breaking of this global symmetry gives rise to a Goldstone boson, called the Majoron, and to a Majorana mass term for the neutrino. Lepton number nonconserving double beta decay can now occur with the emission of two electrons and a Majoron:

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + \chi \ . \tag{3}$$

As a result of the beautiful experiment of Elliott, Hahn, and Moe[1], the two-neutrino decay of ^{82}Se has now been seen in the laboratory with a half-life of order 10^{20} years. This confirms the earlier results of geochemical experiments[5] in which one observes the daughter nucleus rather than the decay electrons. The agreement between both methods in this case lends credence to geochemical measurements yielding a half-live of order 10^{21} years for the decay of ^{130}Te.

No-neutrino decay has not been seen and the best limit on its lifetime comes from experiments searching for the decay of ^{76}Ge:

$$\tau_{\frac{1}{2}}(^{76}Ge; 0\nu) \geq (5-10) \times 10^{23}\text{years} \ . \tag{4}$$

As far as the Majoron mode is concerned, one experiment claimed to see it, but several subsequent ones failed to do so and set limits on its strength below that of the original experiment.

Given this state of affairs, it seems to me that three major questions need to be addressed in the field of double beta decay:

(1) Now that a lifetime of order 10^{20} years has been observed in the laboratory, what should be the next step?

(2) How much further can the limit on the no-neutrino half-life be pushed?

(3) Does the Majoron really exist?

In order to develop answers to them, I shall describe briefly the nuclear and particle physics contexts of the phenomena, the kinematical features of the different modes, and the problems associated with calculations of nuclear matrix elements.

2 NUCLEAR PHYSICS SETTING

Because of the pairing force, even-even nuclei tend to lie lower in energy than neighboring odd-odd nuclei. It can therefore happen for a given triad of nuclei (A,Z), (A,Z+1), and (A,Z+2) where the atomic weight A and the atomic number Z are both even numbers that the central member (A,Z+1) is heavier than the other two and that the first one (A,Z) is heavier than the third one (A,Z+2). Transitions from the first to the central member are forbidden by energy conservation, but transitions from the first to the third member are allowed. Thus they can occur as second-order effects of the same interaction which in first order gives rise to single beta decay.

The ground-states of even-even nuclei have zero spin and positive parity, and they characteristically have an excited state of spin 2 and positive parity about 500 keV above them. Ground-state to ground-state transitions are therefore of the type:

$$0^+ \rightarrow 0^+ \tag{5}$$

and transitions to the excited state involve a spin change of two units:

$$0^+ \rightarrow 2^+ \ . \tag{6}$$

Typically some 2 to 3 MeV of energy are released in these processes. Examples of double beta decay parent and daughter nuclei are given in Table 1 below.

TABLE 1 Examples of Double Beta Decay Nuclei[5].

Parent Nucleus	Daughter Nucleus	Energy Release Q (MeV)	Comment
^{48}Ca	^{48}Ti	4.27	Largest energy release
^{76}Ge	^{76}Se	2.04	Series of 0ν expts
^{82}Se	^{82}Kr	3.00	Geochem. and Lab.
^{100}Mo	^{100}Ru	3.03	New expts
^{130}Te	^{130}Xe	2.53	Geochemical
^{128}Te	^{128}Xe	0.87	Geochemical
^{136}Xe	^{136}Ba	2.48	New expts, TPC
^{150}Nd	^{150}Sm	3.37	
^{232}Th	^{232}U	0.86	
^{238}U	^{238}Pu	1.15	New expt

3 PARTICLE PHYSICS REQUIREMENTS

The two-neutrino decay mode is expected to occur in the standard model as a second-order effect of the beta decay Hamiltonian and it imposes no special requirements on the properties of the neutrino; it will occur irrespective of whether the neutrino is a Majorana or a Dirac particle and irrespective of whether it has mass or not. No-neutrino decay, on the other hand requires physics beyond the standard model.

If no-neutrino decay is to occur, then lepton number cannot be conserved: in fact it must change by two units because the final state contains two leptons where the initial state contains none. Either there must exist some entirely new interaction of unknown strength which causes this breakdown, or one of the neutrinos coupling to the electron in the standard weak current must be a Majorana particle. In the latter, and most often examined, case no-neutrino decay comes about through the exchange of a Majorana neutrino between two neutrons inside the nucleus. Given the (V-A) nature of the standard weak current, the neutrino must be able to flip its helicity in its passage, real or virtual, from one neutron to the other; this requires either that the neutrino have mass, or that there exist some small admixture of (V+A) currents in the beta decay interaction, or both. From a gauge-theoretic point of view, both of these helicity flip mechanisms require spontaneous breaking of the gauge symmetry by means of a neutrino mass matrix.

This condition on the breaking of gauge theories arises from the requirement of good high energy behavior for the amplitude for two electrons to transform into two negatively charged gauge bosons via the mechanism of neutrino exchange[6]:

$$e^- e^- \to (W_a)^- (W_b)^- \quad \text{via neutrino exchange} \; . \tag{7}$$

It should be recalled that good high energy behavior is at the heart of renormalisability, and that it is brought about by cancellations between different types of diagram and by various algebraic conditions; in all cases it leads to relations between coupling constants of the same kind as may be imposed by symmetry groups, for example SU(2) and SU(3). The process in eq.(7) is a factor of all double beta decay transitions, the transitions being completed when the gauge bosons are hooked on to up- and down-quarks.

When the currents coupling to W_a and W_b both have the same helicity, then the amplitude for eq.(7) is proportional to the mass term in the neutrino propagator and it automatically vanishes when the neutrino mass vanishes. When the two currents have opposite helicities, the resulting amplitude for eq.(7) is, in general, quadratically divergent, and it will lead to bad high energy behavior unless the leading divergence is exactly cancelled by another diagram or by some other condition. The only other diagram which could bring about a cancellation is one in which the electrons couple directly to a doubly charged boson; the existence of such a boson is not required in present-day phenomenology and it would have the unattractive feature of leading to quarks with charges $+\frac{5}{3}$ or $-\frac{4}{3}$ in the spectrum of elementary fermions. We therefore prefer the alternative of another condition.

Such a condition is tantamount to saying that the neutrino which couples to the electron e^- in the current of W_a must be orthogonal to the charge conjugate of the neutrino coupling to e^- in the current of W_b. This orthogonality is in keeping with the algebraic structure of gauge theories, and it yields an algebraic condition which ensures the vanishing of the amplitude for eq.(7) when all neutrinos which contribute to it either have zero mass, or have exactly the same nonzero mass[6]. Thus it is mass differences, arising as they do from spontaneous breaking of gauge symmetries, that yield nonzero amplitudes for eq.(7) and hence for no-neutrino double beta decay.

At first sight, it might seem that this requirement of nonzero and distinct masses for neutrinos might be evaded were we to opt for an entirely new $\Delta L = 2$ interaction as the mechanism for no-neutrino double beta decay in place of neutrino exchange. However such a mechanism, while not postulating a neutrino mass ab initio, still leads to an induced mass through second-, and higher-order diagrams which transform a neutrino ν_L into its charge conjugate $(\nu_L)^C$ and are the equivalent of a Majorana mass term. On dimensional grounds, this mass is expected to be small, being of order:

$$\delta m_{\mathrm{Majorana}} \approx g_{\beta\beta}(G_F)^2 E^5 \quad , \tag{8}$$

where $g_{\beta\beta}$ is the strength of the new interaction in dimensionless units and is of order 10^{-3} or less, G_F is the Fermi constant for beta decay, and E is a virtual energy characteristic of second-order weak processes in nuclei. Taking $E \approx 100 MeV$, which corresponds to a mean separation between nucleons of 1-2 fermi and is probably optimistic, we find that

$$\delta m_{\mathrm{Majorana}} \approx g_{\beta\beta} 10^{-6} eV \approx 10^{-9} eV \quad . \tag{9}$$

This value is much smaller than the $\mathrm{K_L} - \mathrm{K_S}$ mass difference, also a second-order weak effect and approximately equal to 10^{-5} eV, because of the very weak strength of the $\Delta L = 2$ interaction.

4 KINEMATICAL FEATURES OF THE DECAY MODES

The three types of double beta decay have different kinematical features, and these differences may be used to distinguish between them at the observational level. In this section we discuss the properties of phase space, energy spectra, and angular distributions[3].

Because leptons are so much lighter than nucleons, and because the energy released in double beta decay is very small compared with the rest-mass of the parent nucleus, we treat the nucleus as being infinitely heavy and ignore, in most cases, the recoil of the daughter. Two-neutrino decay then has a four-body phase space corresponding to the two electrons and two anti-neutrinos in the final state. In terms of the energy release Q, the four-body phase space behaves roughly like the tenth to eleventh power of Q, and so the the half-life for two-neutrino decay is inversely proportional to to this factor:

$$\frac{1}{\tau_{\frac{1}{2}}(2\nu)} \propto Q^{10-11} \quad . \tag{10}$$

No-neutrino decay has a two-body phase space corresponding to the two electrons that populate the final state, but it has an additional factor arising from the integral over the virtual neutrino exchanged between nucleons inside the nucleus. Crudely speaking each of these factors is proportional to the fifth power of an energy, Q for the phase space and the mean neutrino energy $\langle E_\nu \rangle$ for the integral:

$$\frac{1}{\tau_{\frac{1}{2}}(0\nu)} \propto \langle E_\nu \rangle^5 Q^5 . \tag{11}$$

The ratio of two-neutrino to no-neutrino lifetimes is then roughly proportional to:

$$R = \frac{\tau_{\frac{1}{2}}(2\nu)}{\tau_{\frac{1}{2}}(0\nu)} \propto \left(\frac{\langle E_\nu \rangle}{Q} \right)^5 . \tag{12}$$

For a mean virtual neutrino energy of 50 MeV and an energy release of 3 MeV, the ratio of lifetimes is of order 10^6. This means that, were all other factors, for example coupling constants, equal, the no-neutrino mode would be about a million times faster than the no-neutrino mode. It follows that no-neutrino decay is sensitive to very small lepton-nonconserving parameters. In fact this argument is the origin of our choice $g_{\beta\beta} \approx 10^{-3}$ in the previous section.

Majoron decay involves a three-body final state and so the phase space for it lies somewhere between those for the no-neutrino and two-neutrino modes. Thus it varies as the eighth power of Q, or thereabouts.

A useful tool for distinguishing between these modes is the spectrum of events plotted as a function of the sum of the energies of the electrons. For two-neutrino decay the spectrum will be a continuous, broad distribution with its peak just below the mid value Q/2 of the energy sum; this corresponds to an approximately equal sharing of the energy release between the electrons and anti-neutrinos once the rest-mass of the electrons has been taken into account. For no-neutrino decay the electrons carry off the entire energy release and so the sum of their energies must be constant; the resulting spectrum is a spike at the end-point Q. For Majoron decay, angular momentum conservation requires the two electrons to travel in opposite directions (see below), and so the Majoron will tend to have a 'soft' momentum; the peak of the sum spectrum for this mode will therefore be shifted beyond the mid-point Q/2 and will fall closer to the end-point. It is the shape of this last spectrum that provides the basis for deciding whether the Majoron exists.

We now turn to the question of the angular correlation between electrons for the different decay modes. The essential point in this discussion is that ground-state to ground-state transitions are all of the type $0^+ \to 0^+$ and so the leptons in the final state must have zero total angular momentum. Similarly in the approximation of an infinitely heavy nucleus, the leptons must have zero total linear momentum.

In the standard electroweak model electrons have negative helicity and anti-neutrinos positive helicity. If we consider a colinear decay configuration in which all the leptons travel along, or anti-parallel to, a given direction, then the conservation of angular momentum in that direction amounts to the conservation of spin components in the given direction.

Now take the case of two-neutrino decay: we must arrange the colinear configuration in such a way that linear momentum and spin are both conserved in the final state. The only way to do this is to have the two electrons be emitted with roughly equal and opposite momenta, and likewise for the two anti- neutrinos. Therefore the angular correlation must be of the form:

$$A_{2\nu}(e_1, e_2) \propto (1 - \beta_1\beta_2 \cos\theta_{12}) \ , \tag{13}$$

where θ_{12} is the angle between the electrons and β_i(i=1,2) are the speeds of the electrons in units of the speed of light.

For no-neutrino decay, we must distinguish between the two phenomenologically allowed mechanisms, namely the direct neutrino mass mechanism and the interference of left- and right-handed leptonic currents. In the former case the electrons both have the same helicity and so the back-to-back configuration conserves linear and angular momentum at the same time. In the latter case, the electrons will have opposite helicities, and one has to appeal to the nuclear recoil to conserve both types of momenta: angular momentum requires the electrons to be parallel and linear momentum requires the nucleus to recoil in the opposite direction. The corresponding angular distributions are:

$$A_{0\nu,\mathrm{mass}}(e_1, e_2) \propto (1 - \beta_1\beta_2 \cos\theta_{12}) \ , \tag{14}$$

and

$$A_{0\nu,\mathrm{LR}}(e_1, e_2) \propto (1 + \beta_1\beta_2 \cos\theta_{12}) \ , \tag{15}$$

respectively. An important experimental question to ask regarding the LR angular distribution of eq.(15) is:

DO TPC AND SANDWICH DETECTORS HAVE AN INHERENT BIAS AGAINST THIS TYPE OF CORRELATION ?

The Majoron has zero spin and in this decay mode the electrons again have the same helicity. Thus the electrons must again be in a back-to-back configuration as in eq.(14) above.

It is also interesting to consider the ground-state to spin 2^+ excited state transition and the angular distribution for it in the various possible cases. In the case of two-neutrino decay, we can conserve linear momentum and angular momentum in a transition in which the ground-state of the parent decays to the $J_z = 2$ component of the excited state by means of a colinear configuration in which the two electrons go off in the z-direction and the two anti-neutrinos go off in the opposite direction. Given the standard helicity assignments, the total J_z of the leptons will be (-2) and just balance the spin component of the daughter nucleus. Not all transitions involve the same values of the z-components of angular momentum, and so the angular correlation will be, on average:

$$A_{2\nu}(e_1, e_2; 0 \rightarrow 2) \propto (1 + \frac{1}{3}\beta_1\beta_2 \cos^2\theta_{12}) \quad . \tag{16}$$

In the case of no-neutrino decay, we must again consider the two mechanisms separately. The mass mechanism gives rise to two electrons which are in a relative S-state and so the only leptonic angular momentum is the spin angular momentum. This is not sufficient to balance the nuclear spin change of two units, and therefore the mass mechanism cannot engender transitions to the excited state. The left-right interference mechanism, on the other hand, gives rise to two electrons in a relative P-state, and when coupled with the spins of the electrons, this can balance the nuclear spin change. It follows that the detection of a no-neutrino transition to the spin 2 excited state of the daughter nucleus would be an unambiguous signal for decay through the interference of left- and right-handed leptonic currents[7].

5 NUCLEAR MATRIX ELEMENTS

Having disposed of the kinematics of double beta decay, we must now deal with the heart of the problem, namely the nuclear matrix element. It is always calculated in second-order perturbation theory and takes the general form[3]:

$$M = \sum_m \frac{\langle f \mid H_\beta \mid m\rangle\langle m \mid H_\beta \mid i\rangle}{E_m - E_i} \tag{17}$$

where i, m, f, denote the initial, intermediate, and final nuclear states respectively, and H_β represents the standard operators for single beta decay. The energy denominator is given by:

$$(E_m - E_i) = W_{mi} + E_{\nu_i} + E_{e_j} \tag{18}$$

where W_{mi} denotes the energy difference between the ground-state of the parent nucleus and the state m of the intermediate nucleus, E_{ν_i} is the energy of the neutrino in the intermediate state, and E_{e_j} is that of the electron.

What we would like to argue in this section is that the matrix elements for ground-state to ground-state 2ν transitions are likely to be small, and likewise for 0ν transitions. Many years ago, Henry Primakoff and I argued that in general the single beta decay matrix elements appearing in eq.(17) above are of the 'allowed but hindered' variety, and that the number of intermediate states is limited[8]. We then estimated a central value for the double beta decay matrix element of 0.1, but because of the crudeness of our argument, we gave ourselves leeway of an order of magnitude in both the increasing and the decreasing directions. Today, it is possible to make more sophisticated arguments but they have the same tendency as our original one.

It is common practice to replace the energy difference between nuclear states, W_{mi}, by an average value so that the sum over the intermediate nuclear levels can be performed by closure. In the case of no-neutrino decay this seems to be a reasonable approximation because the typical energy of the exchanged neutrino is of order 50-100 MeV (the inverse of a typical separation between neutrons), whereas the typical value of W_{mi} is of order 5-10 MeV. In the case of two-neutrino decay the use of closure is open to question and several calculations have been made without it. For our purposes, however, we make use of it and replace the entire energy denominator by an average:

$$\langle E_m - E_i \rangle = \langle W_{mi} \rangle + \frac{Q}{2} + m_e c^2. \tag{19}$$

In the usual allowed approximation for beta decay, the operators H_β take the standard forms for Fermi and Gamow-Teller transitions:

$$\begin{aligned} H_\beta &= [\sum_{\text{nucleons}} \tau_k^+] = T^+ \qquad \text{Fermi} \\ H_\beta &= [\sum_{\text{nucleons}} \tau_k^+ \sigma_{\mathbf{k}}] \qquad \text{Gamow-Teller}\ . \end{aligned} \tag{20}$$

The Fermi operator is the raising operator for total nuclear isospin, while the Gamow-Teller operator is a mixed spin-isospin operator of the type that occurs in the supermultiplet scheme of Wigner. In order to analyse the properties of these operators, let us briefly review the role of isospin in nuclei.

I shall use the particle physics convention and define the third component of isospin to be:

$$T_3 = \frac{1}{2}(Z - N) \tag{21}$$

where Z is the number of protons and N is the number of neutrons. For heavy nuclei of the kind that undergo double beta decay the number of neutrons exceeds that of protons, and the ground-state is assigned the smallest total isospin compatible with the third component, namely

$$T = \frac{1}{2}(N - Z) \qquad (N \geq Z) \tag{22}$$

We can now construct the following table of isospins for the triad of nuclei taking part in double beta decay.

Table 2. Ground State Isospin Assignments.

Nucleus	Ground-State Isospin
Initial (A,Z)	$\frac{1}{2}$(N - Z)
Intermediate (A,Z+1)	$\frac{1}{2}$(N - Z) - 1
Final (A,Z + 2)	$\frac{1}{2}$(N - Z) - 2

It follows that ground-state to ground-state transitions involve a change of two units of isospin:

$$T_i - T_f = 2 \tag{23}$$

This is an important observation which will have specific ramifications for double beta decay. For future reference we note that the intermediate and final nuclei (A,Z+1) and (A,Z+2) do have states with isospin $\frac{1}{2}(N - Z)$; they are both excited states and have the same isospin, spin, and parity as the ground-state of (A,Z). They are known as the single and the double isobaric analogue states respectively.

Turning to the double Fermi matrix element for two-neutrino decay,

$$M_F(2\nu) = \frac{1}{\langle E_m - E_i \rangle} \sum_m \langle f \mid T^+ \mid m \rangle \langle m \mid T^+ \mid i \rangle \tag{24}$$

we find that it involves the total isospin raising operator acting twice, first upon the initial state and then upon the intermediate state. Now the raising operator has the property that it increases the third component T_3 eigenvalue by one unit without changing the total isospin T. Therefore th states m and f must have the same isospin as i for the matrix element to be nonzero. In other words the Fermi operator would like to transform the ground-state of the initial nucleus into the double isobaric analogue state of the daughter, rather than the ground-state. Since the ground-state of the daughter nucleus has two units fewer total isospin than that of the daughter, we conclude that, to the extent that isospin is a good quantum number, the corresponding matrix element must vanish:

$$M_F(2\nu; \mathrm{gs} \rightarrow \mathrm{gs}) = 0 \ . \tag{25}$$

What about the double Gamow-Teller matrix element

$$M_{GT}(2\nu) = \frac{1}{\langle E_m - E_i \rangle} \sum_m \langle f \mid \sum_k \tau_k^+ \sigma_k \mid m \rangle . \langle m \mid \sum_l \tau_l^+ \sigma_l \mid i \rangle \ \ ? \tag{26}$$

Should there be a good symmetry scheme in which the operators $\sum \tau^+ \sigma$ belong to the set of generators of the symmetry algebra, then M_{GT} will vanish when the states i and f belong to different representations, just as happens in the case of the Fermi matrix element.

One candidate for such a theory is the Wigner supermultiplet theory, which is based upon the embedding of the direct product of spin and isospin in the group SU(4), and which is generated by the operators:

$$\tau_i, \sigma_j, (\tau_l \sigma_k) \qquad \text{for } i,j,k,l = 1,2,3 \quad . \tag{27}$$

The $J = 0, T$ ground-state of (A,Z) will, in general, belong to a different SU(4) representation than the $J = 0, T-2$ ground-state of (A,Z+2), and so M_{GT} will vanish. Unfortunately SU(4) is not such a good symmetry, and this argument, though illustrative, is not a very strong one.

To demonstrate this point, consider a commutation rule in the algebra, namely:

$$\sum_l [\tau^+ \sigma_l, \tau^+ \sigma_l] = 3(2T_3) = -3(N-Z) \quad . \tag{28}$$

Take the expectation value with respect to the ground-state and insert a complete set of states (that is, the set of all states in the same representation as the ground-state) to yield the sum rule:

$$\sum_m |\langle m | \tau^+ \sigma | 0^+ \rangle|^2 - \sum_n |\langle n | \tau^- \sigma | 0^+ \rangle|^2 = 3(N-Z) \quad . \tag{29}$$

The first term refers to β^- decays from the ground-state and the second to β^+ decays. It turns out that giant Gamow-Teller resonances plus low- lying $J^P = 1^+$ states do not exhaust the sum rule, but fill only about 60% of it. This indicates that the sum rule is not well satisfied, and that SU(4) is not a very good symmetry. A phenomenological way of dealing with this problem is to renormalise the axial vector coupling constant g_A down from its value of 1.26 in neutron decay to a value of 1 for heavy nuclei.

As indicated by this discussion, we must turn to more complicated schemes in order to evaluate the double Gamow-Teller matrix element. Since it gives the dominant contribution to the two-neutrino decay rate even though it is likely to be 'suppressed', we shall return to the subject below. For future purposes we factorise the rate for two-neutrino decay into a product of the square modulus of the matrix element times a factor arising from phase space and Coulomb corrections:

$$\frac{1}{\tau_{\frac{1}{2}}(2\nu; 0^+ \to 0^+)} = |M_{GT}|^2 \, G_{GT}(A,Z) \quad . \tag{30}$$

In the case of no-neutrino decay we use closure over the intermediate nuclear states and integrate over virtual neutrino energies to obtain two matrix elements analogous to the double Fermi and Gamow-Teller matrix elements for the two-neutrino case[5]:

$$
\begin{aligned}
M_F(0\nu) &= \langle f \mid [\sum_{k,l} \frac{(\tau_k)^+(\tau_l)^+}{r_{kl}}] \mid i\rangle \\
M_{GT}(0\nu) &= \langle f \mid [\sum_{k,l} \frac{(\tau_k)^+(\tau_l)^+\sigma_{\mathbf{k}}.\sigma_{\mathbf{l}}}{\mathbf{r_{kl}}}] \mid i\rangle \ .
\end{aligned}
\tag{31}
$$

The factor $\frac{1}{r_{kl}}$ is the neutrino propagator in the small mass limit $m_\nu \ll \langle p_\nu \rangle$, the mean momentum of the exchanged neutrino, and it should be replaced by $\frac{e^{m-\nu r}}{r}$ in the large mass case $m_\nu \gg \langle p_\nu \rangle$.

At first sight, one might expect the neutrino propagator strongly to enhance transitions in which the parent neutrons come very close together. Effects of the nuclear potential, however, appear to mitigate this factor. In hard core potentials, nucleons never come closer than $\frac{1}{3}$ fermi (10^{-13}cms), and in other potentials the optimal attraction occurs at a little over 1 fermi. From a shell model point of view, one must also allow for the fact that the more deeply bound nucleons in the inner shells are much less likely to take part in double beta decay than are the 'valence' nucleons in the outer shells.

With limitations like these in mind, Primakoff and I[8] proposed that the neutrino propagator $\frac{1}{r_{kl}}$ in eq.(31) be replaced by an average value, which we took to be the nuclear radius

$$
\langle r_{kl} \rangle \approx R(A,Z) \approx 1.2A^{\frac{1}{3}}\text{fermi} \ . \tag{32}
$$

An immediate consequence is that the no-neutrino matrix elements are directly proportional to the corresponding two-neutrino ones. It follows that the Fermi matrix element should be small for both types of decay, that both decay rates depend only on the one double Gamow-Teller matrix, and that the ratio of the rates for the two decay modes depends only on the kinematical properties of the parent nucleus. This is one reason why measurements of the two-neutrino decay rate are so important for extracting bounds on lepton number violating parameters from no-neutrino decay.

Shell model calculations by Haxton, Stephenson, and Strottman[9] seem to support the proportionality between the two kinds of matrix element, but with a smaller mean separation of about one half the nuclear radius. More recent calculations based on quasi-particles and the random phase approximation[10,11,12], do not support the proportionality and tend to emphasize the short-distance contributions to the no-neutrino matrix element from nearby pairs of neutrons. Thus it becomes a question whether proportionality holds and whether the Fermi matrix element for no-neutrino decay is small.

One way to see how the proportionality might be lost is to expand the propagator in terms of spherical harmonics:

$$\frac{1}{r_{kl}} = \frac{1}{r_>}[\sum_{L,M} d_{L,M} \left(\frac{r_<}{r_>}\right)^L Y_L^M(\Omega_<) Y_L^{-M}(\Omega_>) \ , \qquad (33)$$

where the subscripts $<$ and $>$ refer to the lesser and the greater of the position vectors $\mathbf{r}_k$ and $\mathbf{r}_l$ respectively. From eq.(33) we see that because of the spherical harmonics, the sum over intermediate states must include all spins. By contrast the corresponding sum for the double Gamow-Teller matrix element of two-neutrino decay is limited to $J^P = 1^+$ states, as can be seen from eq.(26).

Thus it is not obvious that there is a simple relationship between the matrix elements for the two modes of double beta decay. Nevertheless we can gain important insights about nuclear wavefunctions from the two-neutrino case.

6 THE SHELL MODEL AND TWO-NEUTRINO DECAY

With this thought in mind, let us look at two-neutrino decay from the viewpoint of the shell model. In its simplest form the shell model consists of a series of energy levels determined by a mean-field, Hartree-Fock potential. The quantum numbers characterizing the levels consist of the principal quantum number n, the orbital angular momentum l which is denoted by the usual spectroscopic notation (l=0,1,2,3,4,5, are represented by s,p,d,f,g,h, respectively), and the total angular momentum j. In the Nilsson model, which we adopt here, we shall concentrate on the angular momentum quantum numbers.

The levels are filled successively in accordance with the Pauli Principle, and neutrons and protons are treated separately. Each shell with total angular momentum j can accommodate 2j+1 particles at most, and since j is always a half-integer, the maximum occupation number is even. A subset of levels which covers many of the double beta decay parent nuclei is shown in Table 3. Since the nuclei with which we shall be concerned contain more neutrons than protons, the last neutron level to be filled in a given nucleus is generally different from that of the last proton level; this is also illustrated in Table 3.

Table 3 Nilsson model energy levels[13] for double beta parent nuclei (in descending order), indicating the levels of the last neutrons and protons to be filled.

Energy Level	Total Number of Particles	Protons Z	Protons Nucleus	Neutrons N	Neutrons Nucleus
$s_{\frac{1}{2}}$	82			82	^{136}Xe
$d_{\frac{3}{2}}$	80			78	^{130}Te
$h_{\frac{11}{2}}$	76			76	^{128}Te
$d_{\frac{5}{2}}$	64				
$g_{\frac{7}{2}}$	58			58	^{100}Mo
$g_{\frac{7}{2}}$	58	54	^{136}Xe		
$g_{\frac{7}{2}}$	58	52	^{130}Te		
$g_{\frac{7}{2}}$	58	52	^{128}Te		
$g_{\frac{9}{2}}$	50			48	^{82}Se
$g_{\frac{9}{2}}$	50			44	^{76}Ge
$g_{\frac{9}{2}}$	50	42	^{100}Mo		
$p_{\frac{1}{2}}$	40				
$f_{\frac{5}{2}}$	38	34	^{82}Se		
$p_{\frac{3}{2}}$	32	32	^{76}Ge		
$f_{\frac{7}{2}}$	28			28	^{48}Ca
$d_{\frac{3}{2}}$	20	20	^{48}Ca		

The dominant contribution to beta decay transitions is usually thought to involve the most loosely bound neutrons and protons, that is those particles located in the last shell to be filled. For ground-state to ground-state transitions this means that two neutrons in the last neutron shell to be filled must transform into two protons in the lowest open proton shell. It is not difficult to see from Table 3 that such transitions will generally involve a change in the orbital angular momentum of the nucleon undergoing it. Now the Gamow-Teller operator can change spin and isospin, but not orbital angular momentum (see eq.(20)). Therefore we expect such transitions to be forbidden or strongly suppressed.

Another property of the Gamow-Teller operator is that it yields greater probabilities for transformations from one spin-orbit partner to another than for transformations within the same shell. In other words, it prefers to transform the state with $j = l + \frac{1}{2}$ into the state with $j = l - \frac{1}{2}$, or vice versa, than to conserve the value of j. This feature together with the orbital angular momentum argument suggests that of all the transitions listed in Table 4 below, only the one involving ^{100}Mo is not likely to be suppressed.

Table 4 Simple-Minded View of Double Beta Decay Transitions.

Nuclear Decay	Shell-Model Transition	Comment
$^{48}Ca \rightarrow ^{48}Ti$	$(f_{\frac{7}{2}})^2 \rightarrow (f_{\frac{7}{2}})^2$	suppressed, not spin-orbit partners
$^{76}Ge \rightarrow ^{76}Se$	$(g_{\frac{9}{2}})^2 \rightarrow (f_{\frac{5}{2}})^2$	suppressed by orbital ang. mom.
$^{82}Se \rightarrow ^{82}Kr$	$(g_{\frac{9}{2}})^2 \rightarrow (f_{\frac{5}{2}})^2$	suppressed by orbital ang. mom.
$^{100}Mo \rightarrow ^{100}Ru$	$(g_{\frac{7}{2}})^2 \rightarrow (g_{\frac{9}{2}})^2$	not suppressed
$^{128}Te \rightarrow ^{128}Xe$	$(h_{\frac{11}{2}})^2 \rightarrow (g_{\frac{7}{2}})^2$	suppressed
$^{130}Te \rightarrow ^{130}Xe$	$(d_{\frac{3}{2}})^2 \rightarrow (g_{\frac{7}{2}})^2$	suppressed
$^{136}Xe \rightarrow ^{136}Ba$	$(s_{\frac{1}{2}})^2 \rightarrow (g_{\frac{7}{2}})^2$	suppressed

It has to be clearly kept in mind that these arguments are over-simplified, and that other effects which blur them must be taken into account. Where levels are close, configuration mixing comes into play and neither the initial state nor the final state can be described as simply as in Table 4 above. There are correlations between nucleons, such as hard-core potentials, which keep them separated by some minimum amount. Pairing forces, which are responsible for the energetic conditions permitting double beta decay, make pairs of like nucleons couple to zero angular momentum. Particle-hole and particle-particle interactions have a significant impact on other aspects of pairing, and quadrupole forces provide interactions, and therefore mixing, between orbits whose angular momenta differ by two units.

All of these factors can distort the simple-minded picture given above, but we expect some qualitative features to survive. Thus we have a general tendency to favor ^{100}Mo as a double beta decay candidate over other nuclei that have been investigated. We shall have more to say about this in the next section.

7 QUASI-PARTICLE RANDOM PHASE APPROXIMATION

The quasi-particle random phase approximation (QRPA) is a sophisticated version of the shell model designed to take account of pairing. It has recently been applied to the calculation of double beta decay by three different groups and the results are encouraging. The groups are: Engel, Vogel, and Zirnbauer (EVZ)[10]; Grotz, Klapdor, and Muto (GKM)[11]; and Civitarese, Faessler, and Tomoda (CFT)[12]; and the interesting aspect of the work, first observed by EVZ, is that when particle-particle interactions are taken into account, two-neutrino double beta decay matrix elements can be made vanishingly small. Unfortunately the actual magnitude of the matrix elements is sensitive to the strength of the new interaction, and so the calculations are not yet definitive. Nevertheless they represent a real advance and one may hope that definitive results are not too far away.

The idea of quasi-particles is based on the principle that the presence of a particle with a set of quantum numbers (+q) is equivalent to the absence of a particle with the opposite set of quantum numbers (-q). This principle allows us to invent a new 'particle' which is a linear superposition of the presence of (+q) and the absence of (-q), and to try to describe a nuclear system in terms of these 'particles'.

To implement the idea, we introduce a set of fermion annihilation operators a_k and creation operators $a_k{}^{\dagger}$ which satisfy the usual anti- commutation rules:

$$\{a_k, a_l\} = \{a_k{}^{\dagger}, a_l{}^{\dagger}\} = 0 \tag{34}$$

and

$$\{a_k{}^{\dagger}, a_l\} = \delta_{k,l} \quad . \tag{35}$$

The index k can be regarded as representing the set of quantum numbers of one orbit in a specific shell model. Now construct a set of quasi-particle operators which incorporate the principle stated above and which obey the same anti-commutation rules as the a_k operators. They are given by:

$$\begin{aligned} \alpha_k{}^{\dagger} &= U_k a_k{}^{\dagger} - V_k a_{-k} \\ \alpha_k &= U_k a_k - V_k a_{-k}{}^{\dagger} \quad , \end{aligned} \tag{36}$$

where k is always positive,and U_k and V_K are real numbers satisfying

$$U_k{}^2 + V_k{}^2 = 1 \quad . \tag{37}$$

The principal virtue of this approach is that the ground-state of the α operators contains correlated pairs of the a particles:

$$\begin{aligned} \alpha_k \mid 0\rangle = \alpha_{-k} \mid \tilde{0}\rangle = 0 \\ \mid \tilde{0}\rangle = [\prod_{k>0} \left(1 + \frac{V_k}{U_k} a_k{}^{\dagger} a_{-k}{}^{\dagger}\right)] \mid 0\rangle \quad . \end{aligned} \tag{38}$$

The chief disadvantage is that the scheme does not automatically conserve particle number, and so one must add a subsidiary condition in order to do so.

In this model, the Hamiltonian is given by

$$H = \sum_k \epsilon_k \alpha_k{}^{\dagger} \alpha_k + g_{pp} \sum_{k,l} (\alpha_k{}^{\dagger} \alpha_{-k}{}^{\dagger})(\alpha_l \alpha_{-l}) + g_{ph} \sum_{k,l} (\alpha_k{}^{\dagger} \alpha_{-k})(\alpha_l{}^{\dagger} \alpha_{-l}) + \ldots \quad , \tag{39}$$

where ϵ_k are the energy levels of the shell model, the g_{ph} term represents particle-hole interactions, and g_{pp} is the particle-particle term introduced by EVZ. This new term causes the two-neutrino matrix element $M_{GT}(2\nu)$ of eq.(26) to vanish when

$$g_{pp} = g_{ph} \quad , \tag{40}$$

and to vary rapidly with g_{pp}. From a symmetry point of view, the vanishing of the matrix element can be associated with a dynamical SU(4) which holds when the two coupling constants are equal.

In their actual calculations, EVZ use a zero range force whereas the other groups, GKM and CFT, use a more realistic particle-particle and particle-hole force. Typical results are displayed in Table 5 below. They indicate that without the particle-particle force the calculated half-lives are much too short; and that with a realistic force it is possible to account for the experimental half-lives of both ^{82}Se and ^{130}Te (see the KM,pp force line of Table 5). The estimated half-life for ^{100}Mo is not much longer than the present limit[14].

Table 5 Comparison of QRPA calculations of two-neutrino half- lives.

Calculation	^{76}Ge 10^{21}yr	^{82}Se 10^{20}yr	^{128}Te 10^{24}yr	^{130}Te 10^{21}yr	^{100}Mo 10^{18}yr
EVZ $g_{pp} = g_{ph} = 0$	0.21	0.11	0.01	0.0024	0.32
EVZ $\alpha_1{}' = -390$	1.3	1.2	0.55	0.22	6
KM $g_{pp} = 0$	0.064	0.027	0.02	0.007	
KM pp force	5.5	1.9	2.0	2.0	
experiment	> 0.3	1 – 2	> 5	1.5 – 2.8	> 3.8

It has been emphasized by Haxton[15] that standard shell model calculations have come within a factor of 2 of the measured lifetime of ^{82}Se, and that with improved capabilities for handling large sets of basis states, the method may yield an accurate computation of all two-neutrino lifetimes.

The QRPA method has also been applied to the calculation of no-neutrino matrix elements[10,11,12]. There appears to be some suppression in this case too, but the matrix element is not as sensitive to the particle-particle interaction strength g_{pp} as is the 2ν one. As a result, the limits on the neutrino mass extracted from bounds on the half-life for no- neutrino decay are not as tight as those obtained in other calculations. Table 6 contains the QRPA bounds and it indicates that the mass limits are about a factor of 3 larger than those obtained from shell model calculations.

Table 6 Bounds on neutrino mass extracted from no-neutrino double beta decay lifetime limits using QRPA.

Lepton Nonconserving Parameter	^{76}Ge	^{82}Se	^{128}Te	^{130}Te
Experimental limit on 0ν half-life(yr)	5×10^{23}	10^{22}	5×10^{24}	1.5×10^{21}
$\langle m_\nu \rangle$ eV				
CFT	≤ 2.5	≤ 8.2	≤ 1.9	≤ 21
EVZ ($g_{pp} = 0$)	≤ 2.3			
EVZ ($\alpha' = -390$)	≤ 8			
EVZ ($\alpha' = -405$)	≤ 10		≤ 2.2	≤ 26
Left-Right Parameters				
CTF $\langle \lambda \rangle \times 10^6$	≤ 3.6	≤ 8.5	≤ 5.5	≤ 24
CTF $\langle \eta \rangle \times 10^8$	≤ 2.8	≤ 9	≤ 1.8	≤ 21

A novel approach to the calculation of double beta decay matrix elements is taken in the work of Ching, Chengrui and Ho, Tsohsiu[16]. These authors represent the effect of the energy denominator in second-order perturbation theory by expanding the decay operators in a series of multiple commutators. They use the Paris potential to describe the inter-nucleon force and calculate matrix elements for both modes of decay in ^{48}Ca. As a result they are able to reduce the two-neutrino matrix element by a significant factor as compared with closure, and they predict a half-life of 6 times the present limit of 4×10^{19} years. In 0ν decay, the Fermi matrix element is smaller than the Gamow-Teller one by a factor of 2-7, and the Gamow-Teller matrix element is about the same as that calculated by means of closure.

8 WHERE DO WE GO FROM HERE?

Having covered the major features of double beta decay at this time, I would like to return to the questions I raised at the beginning. The observation of a lifetime of 10^{20} years in the laboratory is a remarkable achievement, and a most encouraging one. We can greatly admire it and, at the same time, regard it as the starting point for a new program of experimentation on the two-neutrino mode of double beta decay. One obvious step is to look for parent isotopes whose lifetimes are likely to be shorter than that of ^{82}Se, or comparable with it; and the aim of the program should be to measure as many properties of each decay as possible. In addition to the half-life, these would include the energy spectra of single and double electrons, and the angular correlation between the electrons. Measurements of the half-lives of a range of different nuclei should provide us with important insights into the nuclear physics of the phenomenon; and measurements of the other properties will enable us to confirm those predictions of the standard model that are less sensitive to the nuclear physics.

Isotopes which are attractive from this point of view have phase space and Coulomb Factors comparable with, or better than those for ^{82}Se. The energy release must therefore be greater than 2.5 MeV and except for ^{48}Ca the value of Z must be greater than 34. A list of possible parent nuclei is given in Table 7 together with their inverse phase space times Coulomb factors for both two-neutrino and no-neutrino decay. In all cases the existing limits on the two-neutrino half-life are less than 10^{20} years, although in the case of ^{48}Ca they come within a factor of 3 of this value.

For comparison we also show the factors for ^{82}Se and ^{130}Te. It is amusing to note that although the phase space factors for these two nuclei are almost equal (the smaller energy release in ^{130}Te being offset by the larger Z), the half-lives differ by a factor of 20 (see Table 5). This implies that the matrix element for ^{82}Se is about 4 times larger than that for ^{130}Te, a result which may be qualitatively explained by noting that while the former decay involves an orbital angular momentum change of 1 unit for each nucleon, the latter involves a change of 2 units (see Table 4). It may be useful to keep this point in mind when selecting candidates from Table 7.

From a more realistic theoretical perspective, we need to continue pursuing approaches like QRPA[10,11,12] and the use of larger and larger shell model bases[15] in the computation of two-neutrino half-lives. The level structure of ^{48}Ca is well understood, and so it is an interesting nucleus from the computational point of view. ^{100}Mo is attractive because it could well have a large matrix element and a half-life of order 10^{19} years. ^{136}Xe provides a good example of the source also serving as the detector, as happens in the ^{76}Ge experiments; and ^{150}Nd is attractive because of phase space, but its structure may be complicated and calculations on it may be hard to carry out. A systematic comparison of theory and experiment for these lifetimes will be helpful in the computation of no-neutrino matrix elements and the extraction of lepton number violating parameters.

Table 7 Isotopes with inverse phase space and Coulomb factors (IPSC) comparable with those of ^{82}Se. The numerical values are taken from the review article by Doi, Kotani, and Takasugi; figures in square brackets denote powers of 10.

Parent Isotope	Energy Release (MeV)	2ν IPSC (years)	0ν IPSC (years)
^{48}Ca	4.27	2.5[16]	4.1[24]
^{96}Zr	3.35	5.2[16]	4.5[24]
^{100}Mo	3.03	1.1[17]	5.7[24]
^{116}Cd	2.80	1.3[17]	5.3[24]
^{136}Xe	2.48	2.1[17]	5.5[24]
^{150}Nd	3.37	8.4[15]	1.3[24]
^{82}Se	3.00	2.3[17]	9.3[24]
^{130}Te	2.53	2.1[17]	5.9[24]

In choosing the above parent nuclei for two-neutrino decay, we have emphasized large energy releases. As has recently been observed by Turkevich[17], this has the effect of reducing the relative fraction of the no-neutrino mode in the overall decay of a given isotope. Indeed one can see from the arguments given in section 4 on kinematics that the ratio of no-neutrino to two-neutrino decay varies inversely as the fifth power of the energy release; and so one must decrease the energy release if one wants to increase the ratio. The type of experiment for which this consideration may be important is one in which the daughter nucleus is detected and hence the sum of rates for the two decay modes is measured. Clearly it is necessary to balance the need for a relatively large fraction of no-neutrino decay against the longest lifetime accessible to a particular technique.

As far as the search for no-neutrino decay is concerned, the series of experiments on ^{76}Ge has been the most successful, and it is almost at the 10^{24} years limit. Most of these experiments have been performed with anywhere from 100 to 1000cc (5.6 kg) of natural germanium, which contains about 8% of the double beta decaying parent. There is, however, one experiment in the Soviet Union which has used about 200cc of a 90% enriched source, and which has achieved a competitive sensitivity in a much shorter time than the other experiments. As a result, interest in the use of enriched sources has grown considerably, and it is hoped that, by replacing natural sources with similar amounts of enriched ones, the limit on the half-life can ultimately be pushed down to 10^{26} years and the limit on the mass to the level of 0.1 eV.

One question to ask is whether we can do better with another nucleus. The energy release in ^{76}Ge is only 2 MeV, and as is apparent from Table 7, there are several cases with larger energy releases and better phase space factors. The problem will be, however, the wedding of these nuclei with an experimental technique as effective as that used in the germanium detectors. If, for example, the xenon TPC is successful and its energy resolution can be brought up to the level of germanium detectors, then one should explore the use of enriched xenon sources. If other nuclei plus detector combinations cannot be made as sensitive as germanium, then the only way of exceeding the 10^{26} years limit will be to use much larger quantities of germanium than have been used hitherto, and to observe them for much longer times.

In all of these considerations, it should be kept in mind that if the actual neutrino mass lies in the range suggested by the MSW effect[18], namely 10^{-2} to 10^{-4}eV, then we must achieve sensitivities of order 10^{28} years and longer if we are to see the no-neutrino decay.

In conclusion let me say that there is still much to be learned from and about double beta decay. Much more theoretical analysis will be needed before we understand the nuclear physics of the phenomenon, and many more experiments will be necessary before we learn the properties of all of its modes.

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